This test consists of five relays of six questions each. For each set of relay, X_i refers to the answer of question i of relay X. For example, B_4 is the answer to question 4 of relay B.

Relay A – Algebra

1. Consider $f(x) = x^{10} + ax^8 + bx^6 - cx^5 - dx^2 + 360$, where a, b, c, d are natural numbers. According to Descartes' Rule of Signs, what is the minimum number of imaginary roots for $f(x)$?
2. Find the constant term in the expansion of $\left(x^2 + 1 + \frac{1}{x}\right)^{A_1}$.
3. On a farm with chickens (1 head and 2 legs each) and pigs (1 head and 4 legs each), there are 30 heads and A_2 legs. Find the number of pigs on the farm.
4. The first term of a geometric sequence of real numbers is 1. If the sum of the first A_3 terms is S_1, and the sum of the next A_3 terms is S_2, and $\frac{S_2}{S_1} = 1024$. Let a_k be the second integer in the sequence, compute $a_k - k$.
5. Find the number of ordered pairs of integers (x, y) such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{A_4}$.
6. Find the area of the region bound by $|x - 2y| + |x + y| = 6\left\lfloor\frac{A_5}{6}\right\rfloor$

Relay B – Geometry

1. An equilateral triangle and a regular hexagon have the same perimeter. The ratio of their areas can be expressed as $\frac{m}{n}$ for relatively prime natural numbers m, n. Compute $m + n$.
2. ℓ_1 and ℓ_2 are parallel and 3 units apart. A and B are on ℓ_1 and B_1 units apart. C is on ℓ_2 such that ΔABC is isosceles. Find the number of possible locations for C.
3. There are two concentric circles. Chord EF of the larger circle intersects the smaller circle at G and H. If $EG = GH = HF = B_2$, then the area of the annulus is $B_3\pi$.
4. In ΔXYZ, $\cos\angle X = 0.6$ and $XY = B_3$. Find the number of possible integral lengths of YZ such that ΔXYZ exists, but is not unique.
5. In quadrilateral $MNPQ, m\angle N = 60°, m\angle P = 60°, MN = 3, PQ = 5, NP = B_4$. Compute the square of the length of side MQ.
6. A circular sector has perimeter of 24 and area of B_5. Find the sum of all possible value(s) for the radius.

Relay C – Counting and Probability

1. 50 students are taking at least one of Chemistry and Physics. If 35 of the students are taking Chemistry and 30 of the students are taking Physics, how many students are taking both classes?
2. A drawer has C_1 socks, and each sock is either white or red. Two socks are drawn at random without replacement. Find the number of red socks in the drawer if the probability of both being red is as close to 0.5 as possible.

3. An unfair coin is flipped C_2 times. The probability it comes up heads 4 times is the same as the probability it comes up heads 5 times. The probability that the coin comes up heads if it is flipped once can be expressed as $\frac{m}{n}$ for relatively prime natural numbers m, n. Compute $m+n$.
4. The perimeter of a triangle with integer side lengths is C_3. How many such triangles exist such that no two are congruent to each other?
5. $\left\lfloor \frac{C_4}{2} \right\rfloor$ couples go to dinner together, and sit at a round table. If each couple wish to sit together, in how many distinct ways can they be seated? Two seatings are the same if each person has the same person to his/her left and the same person to his/her right.
6. Let N be the sum of the digits of C_5. Adam wishes to walk up a flight of stairs consisting of N steps 2 or 3 steps at a time. How many ways can he do this? (For example, there are 2 ways to go up 5 steps: 2-3 or 3-2, whereas there is only one way to go up 4 steps: 2-2.)

Relay D – Trigonometry

1. Find the fundamental period of $f(x) = \sin\left(\frac{\pi x}{3}\right) + \cos\left(\frac{\pi x}{5}\right)$.
2. Find the number of petals in the polar graph of $r = \cos(D_1\theta)$
3. A remote control car is going down a hallway at a rate of D_2 feet per minute. If the diameter of each wheel is 3 inches, how fast is each wheel spinning in radians per second?
4. The sum of the solutions over $[0, 4\pi)$ for the equation $5\sin^2 x = D_3 \cos x + 1$ equals $D_4\pi$.
5. If $\sin x + \cos x = 1 + \frac{1}{D_4}$, compute $64(\sin x - \cos x)^2$.
6. Let m and n be the tens and units digits, respectively, of D_5. Compute the number of solutions over $[0, \pi)$ for $\cos(mx) = \sin(nx)$.

Relay E – Miscellaneous

1. The six-digit integer $n = X169Y9$ is divisible by 99, where X, Y are digits. If n is as large as possible, compute the value of X.
2. Let $A = \begin{bmatrix} 3 & -E_1 \\ E_1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix}$. Find $\det(2AB^{-1})$.
3. For how many complex z with integer real and imaginary parts is $|z| = E_2$?
4. Compute the area of the triangle with side lengths $\sqrt{E_3}, 5\sqrt{2}$, and $3\sqrt{10}$.
5. The area enclosed in the polar graph $r = \frac{E_4}{2+\cos\theta}$ can be expressed as $\frac{m\sqrt{n}}{q}\pi$ for natural numbers m, n, q, where m and q are relatively prime, and n has no perfect square divisors other than 1. Compute $m+n+q$.
6. Compute the remainder when 20^{E_5} is divided by 72.